Using Flowcharts for Algorithmic Processes

Time
1-2 class periods, depending on culminating project

Level
Grades 9-12
Algebra 1, Algebra 2, Pre-Calculus, Statistics, Calculus

Purpose
The purpose of this lesson is to have students create flowcharts that describe an algorithmic process. Several curricular examples are provided, but the activity can be adapted to many mathematical topics. The main goal is to have students think critically about a process and be able to describe that process accurately in a flowchart. The curricular topic of the flowchart does not need to be overly complicated. Ideally, creation of the flowchart will challenge the students to consider ideas such as communication, efficiency, and completeness.

Overview
This is a one-day (or two-day, depending on how far you take the extensions) class activity in which students are introduced to algorithms and flowcharts using a card sorting algorithm from www.csunplugged.org and working through a sample flowchart (or two). Students then develop and share their own flowcharts describing an algorithm from their curriculum. Suggested areas are provided in the Teaching Notes section.

Student Outcomes

Learner Objectives:
- Students will understand what is meant by algorithm
- Students will create flowcharts that describe an algorithm

Computational Thinking in STEM Skills:
Problem Solving Skills
- Troubleshooting
- Decomposing Problems into Subproblems
- Generating Algorithmic Solutions
- Applying Algorithms
- Reframing Problems into Known/Familiar Problems

This work is supported by the National Science Foundation under NSF grant CNS-1138461. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

Common Core Standards:
This activity can be used with a variety of mathematical content. Regardless of the mathematical topic, the activity makes use of the following:

- Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
7. Look for and make use of structure.

Prerequisites
Activities, or alternative activities, that should have been done before

Background
Creating flowcharts within the curriculum should not occur on the first day of new material. In order for students to successfully create flowcharts for a specific topic, they should be comfortable with the subject matter. For example, if this activity is to be used with the quadratic formula, students should have enough experience with the quadratic formula that they can focus on ensuring that their flowchart properly describes the algorithm.

Teaching Notes
1. Students should work in pairs or small groups.
2. Give students a pack of 10-15 playing cards. Ask them to shuffle the cards thoroughly in order to mixed them up. Provide each group with one of the three sorting algorithms listed in the first handout (selection, bubble, or quick sort).
    a. Note: some groups might finish quickly, while others may take a while to follow the algorithm. Talk to the groups who finish quickly to see what they noticed. Offer another of the algorithms to try.
3. Once students are finished discuss the meaning of the words "algorithm" and "flowchart" as a class.
    a. Note: an algorithm is a set of defined steps or procedure to follow, and a flowchart is a visual representation of that process.
4. Hand out the Incomplete Bubble Sort Flowchart.
5. Explain the types of items in the flowchart.
6. Work as a class to complete the flowchart.
    a. Note: oval shapes are terminators, rectangles are a process to follow, and rhombuses are questions to be asked. A possible completed flowchart is shown in the Completed Bubble Sort Flowchart document. Note that this may not be the most efficient way to describe the flow, and this could be part of the discussion.

This work is supported by the National Science Foundation under NSF grant CNS-1138461. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

7. Optional activity to use before students work on their own flowcharts: Have students follow a completed flowchart using either the Craps flowchart or the Divisors flowchart.
    a. Note: the Craps flowchart can be played with Monopoly money or candy. The purpose is to engage students in following a complicated flowchart properly.
    b. Note: when using the Divisors flowchart, it is suggested that the teacher not reveal what the flowchart is supposed to accomplish (namely, determine all the divisors of the input number). Instead, ask students to follow the algorithm and try to figure out the connection between the inputs and the outputs.
8. Students make their own flowcharts for any specific topic.
    a. These can be done by hand or using open source software such as OpenOffice Draw or free accounts from a flowchart tool like Luicdchart (www.lucidchart.com)
    b. Note: The students' flowcharts should be tested for completeness. For example, if students are to create a flowchart to factor a trinomial with lead coefficient of 1, the flowchart the students create should test for that condition.
    c. Students could be placed in groups of 3 to test each others' flowcharts for validity/reliability.
    d. Additionally, encourage students to think critically about unusual situations such as a negative number under a square root when using the quadratic formula.
    e. Flowcharts can then be shared with the class and discussed.

Pre-class Preparation
Teachers should be familiar with the given flowcharts to anticipate questions.

Materials and Tools
Enough playing cards for each group to have 10 to 15 cards

Assessment
How will the teacher evaluate if the objectives have been met?

Additional Information
Suggested topics for student-created flowcharts:
- process for factoring trinomials
- multiplying binomials
- quadratic formula
- finding the slope between any two points (consider zero slope and undefined slope)
- Determining special quadrilaterals

This work is supported by the National Science Foundation under NSF grant CNS-1138461. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

- graphing trigonometric functions by hand
- Integrating using u-substitution
- Finding the derivative using the chain rule
- Describing modular arithmetic

Handouts begin on following page.

This work is supported by the National Science Foundation under NSF grant CNS-1138461. However, any opinions, findings, conclusions, and/or recommendations are those of the investigators and do not necessarily reflect the views of the Foundation.

Selection Sort:

1. Shuffle the pack of cards.
2. Place them face down on the table in a row.
3. Starting with the card furthest on the left turn one card over at a time to find the minimum card in your set. After looking at the card, turn it face down. Keep track of which card is the minimum.
4. Swap that minimum card with the card in the first position.
5. The first card is now in the correct position, repeat this procedure again, starting from the second card.
6. Repeat this until no more swaps are performed.

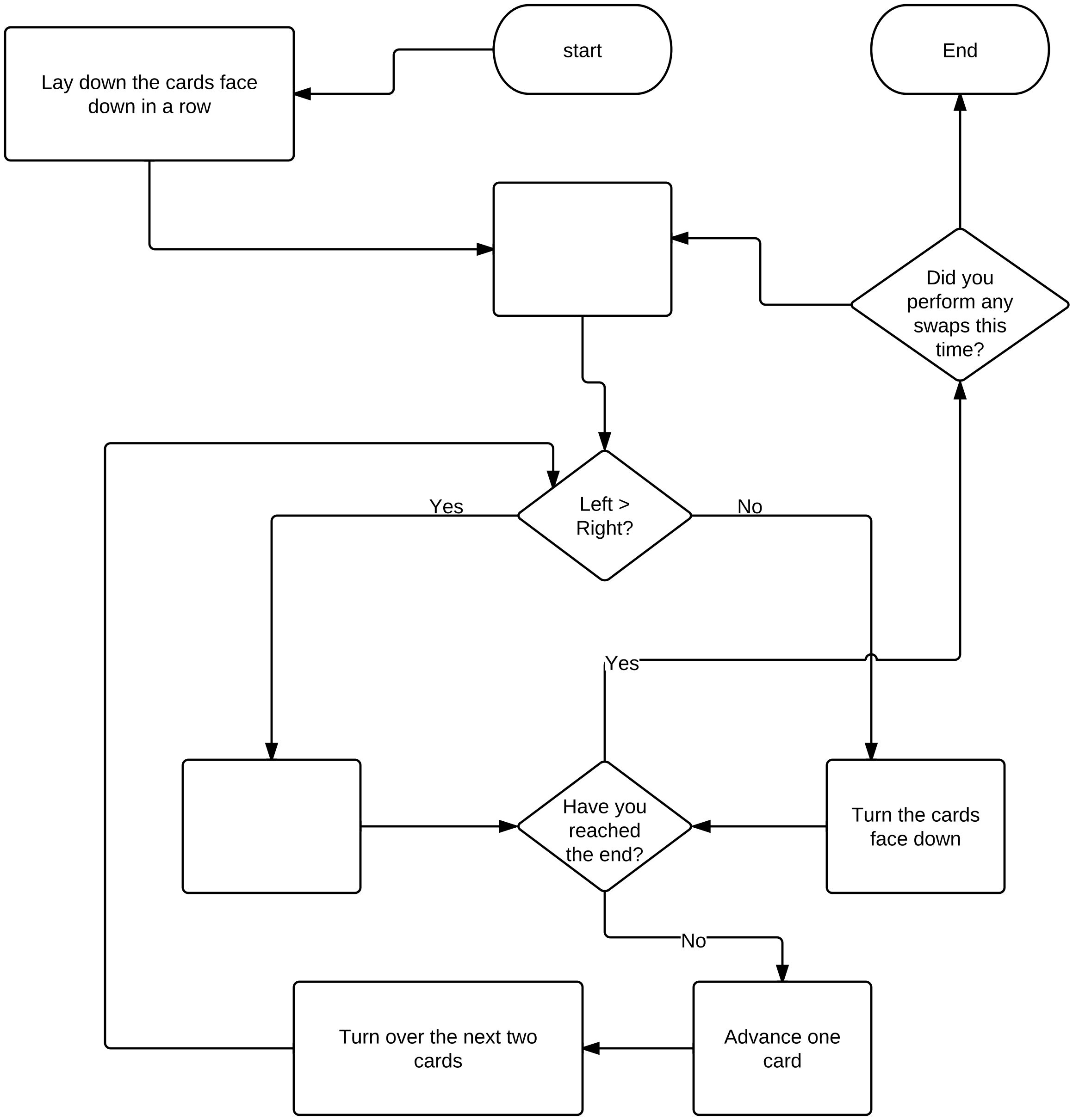
Bubble sort:

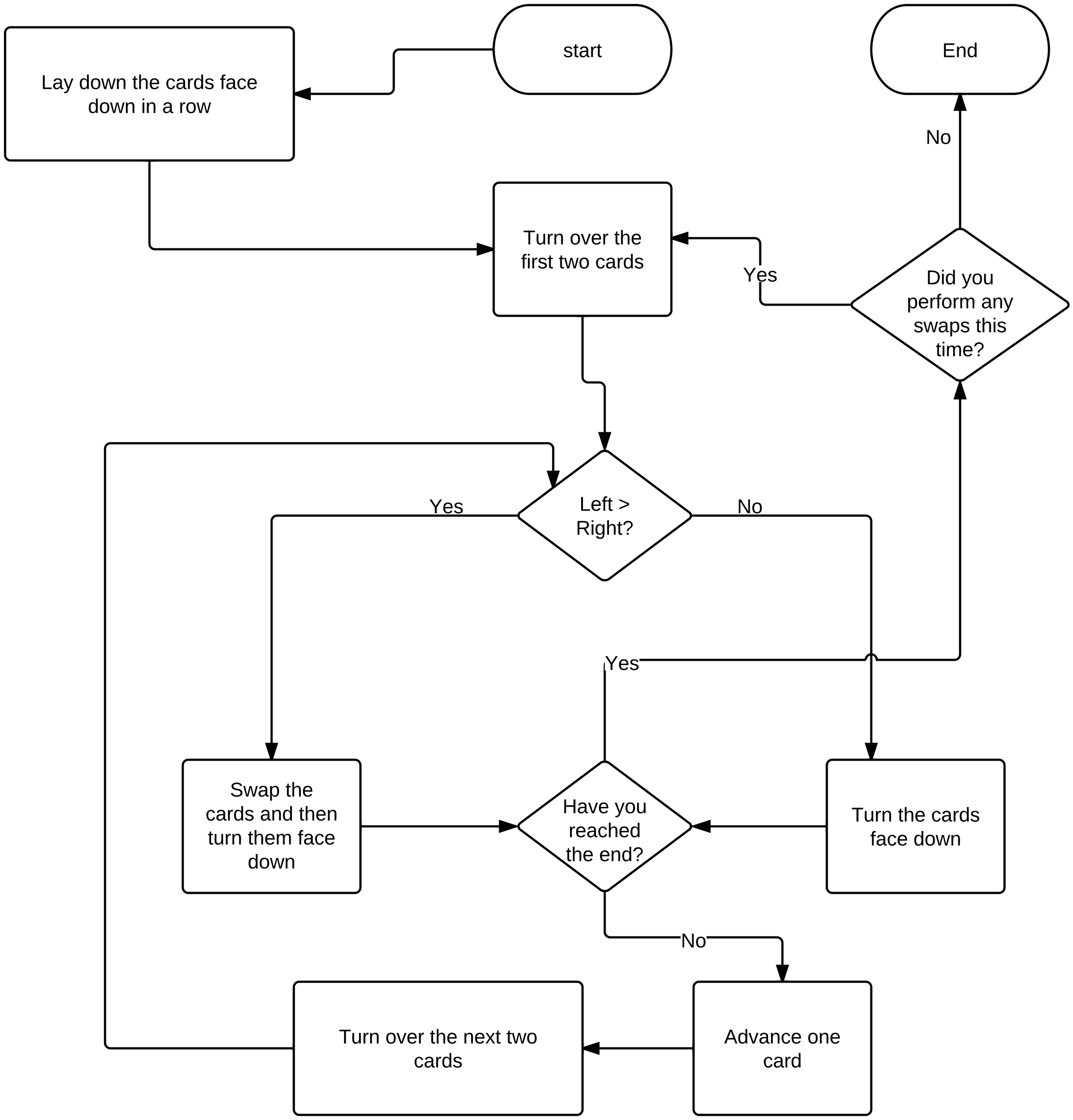
1. Shuffle the pack of cards.
2. Place them face down on the table in a row.
3. Starting with the card furthest to the left, do the following:
4. Look at the current card and the next card. If they are 'out of order' (meaning the current card should be after the next card), swap them.
5. Advance one card (e.g. if you just turned over the first and second card, now turn over the second and third card) and repeat step #4.
6. Continue advancing one card and swapping if necessary until you reach the end of the row of cards.
7. After reaching the end of the row of cards start over at #3.
8. Repeat this until no more swaps are performed.

QuickSort:

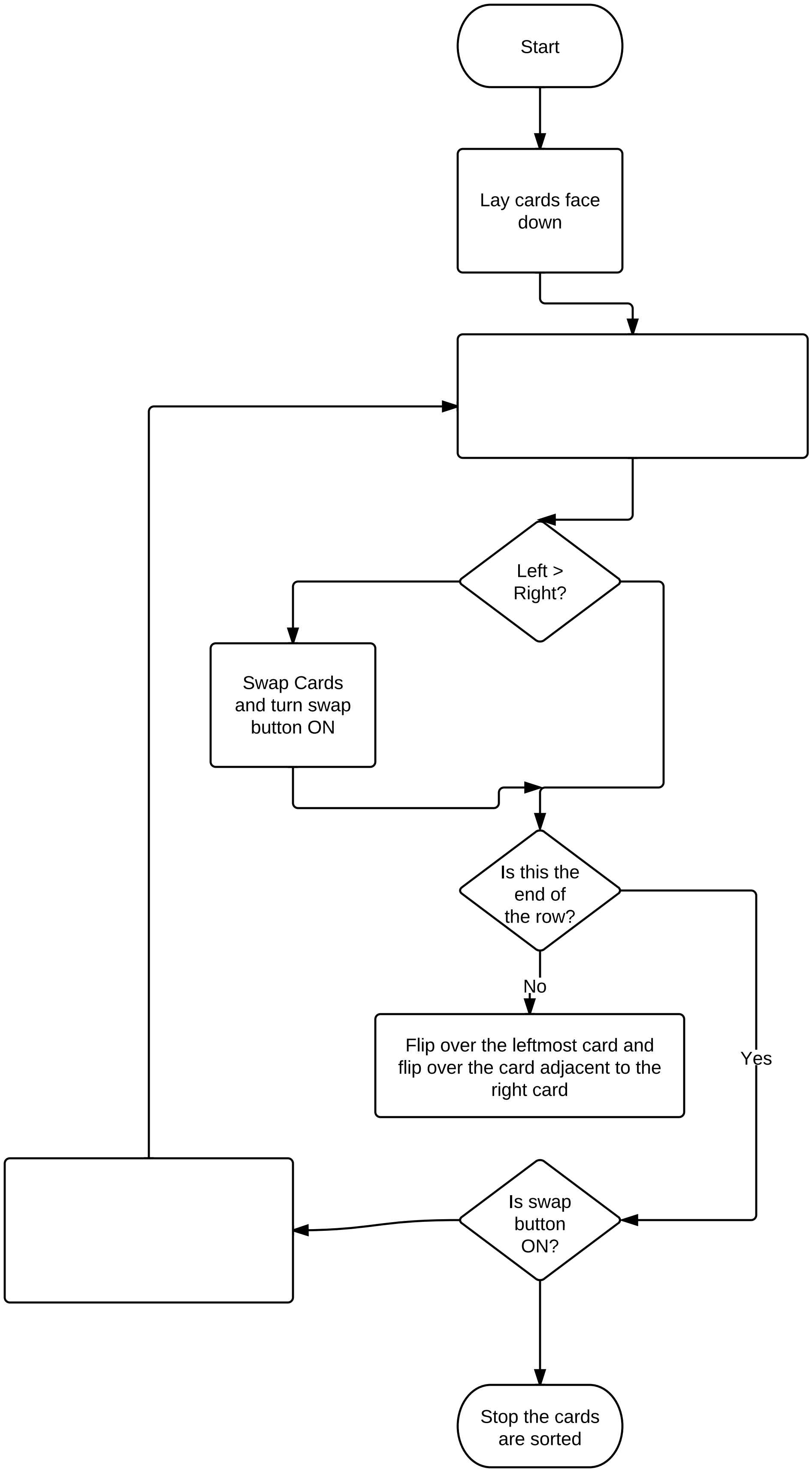
1. Shuffle the pack of cards.
2. Keep the pack of cards face down in your hand.
3. Turn over the top card and place it face up on the table. This is called your pivot card.

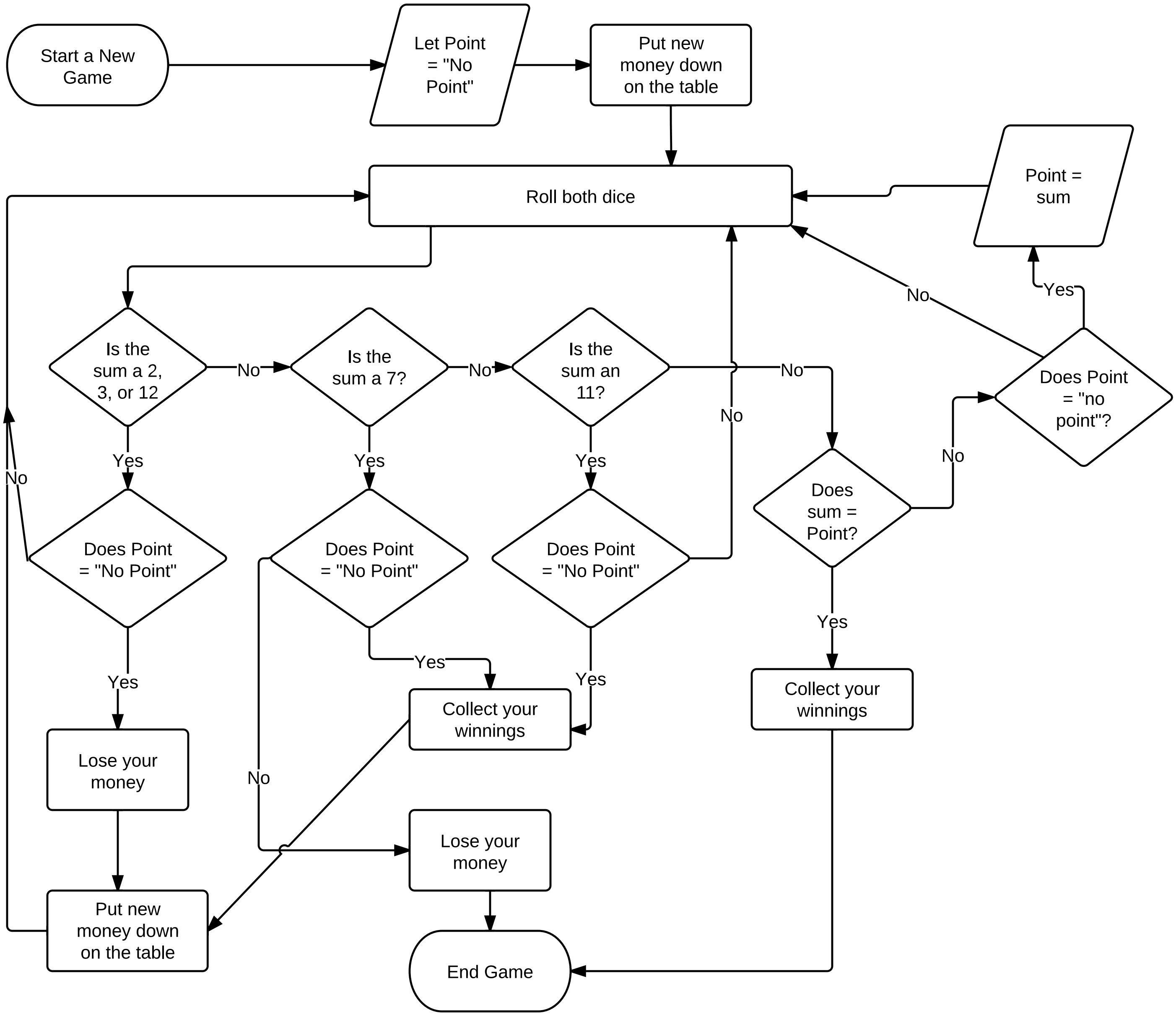
4. Turn over the next card and place it face down to the left or right of your pivot card depending upon whether it is less than or greater than the pivot card.
5. Continue this process through the entire pack.
6. Pick up one of the face down packets of cards and repeat the process starting at #3 using a new pivot card from your face down packet.
7. Repeat this process the remaining face down packets until all cards are face up.

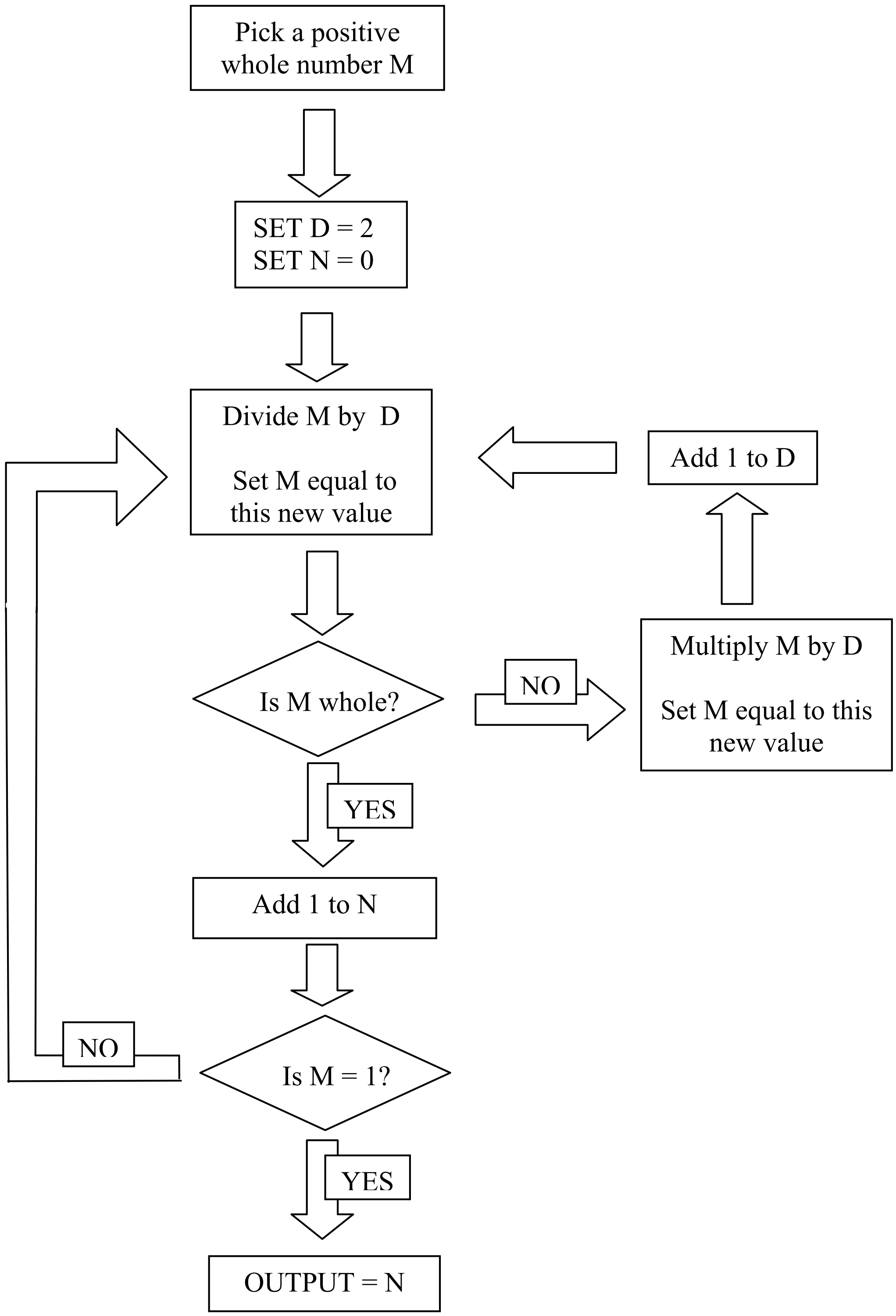

start
End
Lay down the cards face down in a row
Did you perform any swaps this time?
Left > Right?
Yes
No
Yes
Have you reached the end?
Turn the cards face down
No
Turn over the next two cards
Advance one card


start
End
Lay down the cards face down in a row
Turn over the first two cards
Did you perform any swaps this time?
No
Yes
Left > Right?
Yes
No
Yes
Swap the cards and then turn them face down
Have you reached the end?
Turn the cards face down
No
Turn over the next two cards
Advance one card

Make the Bubble Sort flowchart work by adding items such as decisions, instructions and arrows.




Start a New Game
Let Point = "No Point"
Put new money down on the table
Roll both dice
Point = sum
Is the sum a 2, 3, or 12
Is the sum a 7?
Is the sum an 11?
Does Point = "no point"?
Does sum = Point?
Does Point = "No Point"
Does Point = "No Point"
Does Point = "No Point"
Yes
No
Collect your winnings
Collect your winnings
Lose your money
Lose your money
Put new money down on the table
End Game


Pick a positive whole number M
SET D = 2
SET N = 0
Divide M by D
Set M equal to this new value
Add 1 to D
Is M whole?
NO
Multiply M by D
Set M equal to this new value
YES
Add 1 to N
NO
Is M = 1?
YES
OUTPUT = N

Factoring Trinomials with a lead coefficient of 1